B.A.(Prog) Semester-IV (CBCS)
Mathematics: Analysis (DSC)
(Code : 62354443)

M.M. : 100

*Attempt any **two** questions in all.*
All questions carry equal marks.

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Q.1. (*a*) Define neighbourhood of a point, an open set and a closed set. Give an example of each of the following:

(*i*) A nonempty set which is a neighbourhood of each of its points with the exception of one point.

(*ii*) A set which is a neighbourhood of each of its points.

(*iii*) A nonempty set which is neither an open set nor a closed set.

(*iv*) A set which is not a neighbourhood of any of its points.

(*b*) Define limit point of a set. Prove that 0 is the only limit point of the set

$$S = \left\{\frac{1}{n};\ n \epsilon N\right\}$$

(*c*) Prove that if x, y are two positive real numbers, then there exist a positive integer n such that $ny > x$.

Q.2. (*a*) Prove that the set Q of rational number is not a complete ordered field.

(*b*) Show that every continuous function on closed interval is bounded.

(*c*) Show that the function $f(x) = \sin\left(\frac{1}{x}\right)$ is not uniformly continuous on $]0, \infty[$

Q.3. (*a*) Prove that the necessary and sufficient condition for convergence of sequence is that it is bounded and has unique limit point.

(*b*) Prove that a sequence of real numbers converges iff it is a Cauchy Sequence.

(*c*) Show that the sequence $<x_n>$ where $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$ is not convergent.

Q.4. (*a*) State and prove D'Alembert's Ratio Test.

(*b*) Test the convergence of the series

(*i*) $\sum_{n=1}^{\infty}\left(\sqrt{n^3+1} - \sqrt{n^3}\right)$

(*ii*) $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \cdots$

(*c*) (*i*) Prove that a continuous function is Riemann integrable.

(*ii*) Compete $L(p, f)$ and $U(p, f)$ for the function f defined by

$f(x) = x^2$ on $[0, 1]$ and

$p = \left\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\right\}.$

Also state the necessary and sufficient condition for Riemann Integrability.